

Appendix A

Properties of Exponential Integrals

The exponential integral function, $E_n(x)$ is defined for any $n \geq 0$ as follows:

$$E_n(x) = \int_1^\infty \exp(-xz) z^{-n} dz.$$

Some useful properties of this family of functions follow.

$$\begin{aligned} E_0(x) &= \frac{\exp(-x)}{x}. \\ E_n(x) &= \int_z^\infty E_{n-1}(x) dx. \\ \frac{\partial}{\partial x} E_n(x) &= -E_{n-1}(x). \\ E_n(x) &= \frac{1}{n-1} \{ \exp(-x) - xE_{n-1}(x) \}. \end{aligned}$$

Note that the above recursion formula does not enable one to compute the E_1 function from the E_0 function. A FORTRAN subroutine for the E_1 function is given on the next page.

```

function e1(y)
C
C      THIS IS THE E1 FUNCTION THAT ARISES IN TRANSPORT THEORY
C
data c0,c1,c2,c3,c4,c5/-0.57721566,0.99999193,-0.24991055,
1          0.05519,-0.00976004,0.00107857/
data a1,a2,a3,a4/8.5733287401,18.0590169730,8.6347608925,
1          0.2677737343/
data b1,b2,b3,b4/9.5733223454,25.6329561486,21.0996530827,
1          3.9584969228/
C-----
C      DEFINE FUNCTION TO BE SYMMETRIC ABOUT ZERO
C      BY USING ABSOLUTE VALUE OF ARGUMENT
C-----
x = abs(y)
C-----
C      FIRST REPRESENTATION FOR X < 1
C      SECOND REPRESENTATION FOR X > 1
C-----
if (x.lt.1.0) then
  e1 = c0-log(x)+x*(c1+x*(c2+x*(c3+x*(c4+x*c5))))
else
  t = (a4+x*(a3+x*(a2+x*(a1+x))))/(b4+x*(b3+x*(b2+x*(b1+x))))
  e1 = t*exp(-x)/x
end if
C-----
C      CALCULATION COMPLETE
C-----
return
end

```